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ABSTRACT

This paper is designed to provide teachers with more guidance on how to help students become proficient in the recall of basic number facts. Activities and practices are placed in a sequence designed to aid all instructors. The learning of number facts is divided into three stages: (1) the development of basic number facts: (2) establishing number facts: and (3) the memorization of basic number facts. The position adopted is that there is a need for children to have the ability to recall basic number facts readily. This paper confines itself to whole number operations in addition, multiplication, subtraction, and division. Although this material was originally designed for elementary teachers, these materials could be adapted to junior and senior high school use where drill and refreshment in basic number facts is necessary by modifying activities, problem activities, and vocabulary. (MP)

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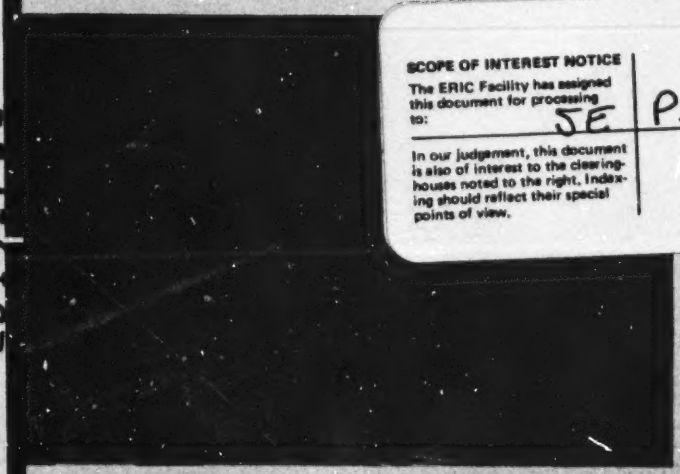

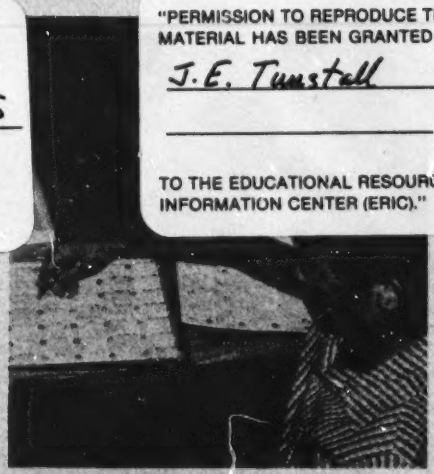

THE LEARNING OF NUMBER FACTS IN THE PRIMARY SCHOOL

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This paper grew out of a need expressed by mathematics advisory teachers for a statement which would provide teachers with more guidance than was given in the Program in Mathematics *on how to help pupils become proficient in the recall of basic number facts. These advisory teachers had identified a range of activities and practices being used by classroom teachers. What they saw as being required was a way of looking at the learning of number facts which placed these activities and practices into a sequence that could assist all teachers.*

The paper looks at the learning of number facts in three stages – development, establishment, memorisation. The suggestions supplement those provided in the Program in Mathematics *where Stages 1 to 4 are concerned mainly with development and establishment, and Stages 5 and 6 with memorisation.*

Introduction

One of the desired outcomes of the *Program in Mathematics* is the development of facility with accuracy in computation. Teachers attempting to help children achieve this outcome are often thwarted in their efforts because of what they consider to be a weakness in the children's ability to recall basic number facts readily.

This paper sets out to look at the way recall of basic number facts may be *developed*, *established* and *memorised*. The stance adopted is that there has been, and will be in the foreseeable future, a need for children in our primary schools to have such recall. Facility with basic number facts makes computation generally more functional and efficient for the learner. This is required in a society where reliance upon calculators in all aspects of living has not yet arrived.

Within this paper 'basic number facts' are confined to:

- addition $0+0=0$ to $9+9=18$
- subtraction $0-0=0$ to $18-9=9$
- multiplication $0\times0=0$ to $9\times9=81$
- division $0\div1=0$ to $81\div9=9$ (with whole number answers only)

It is tempting to think that the ready recall of basic number facts will occur quite naturally without the need on the part of the teacher to organise specific activities for that purpose. Teachers today are aware, however, that a planned program aimed at developing a range of understandings is necessary before memorisation of basic number facts can take place. On the other hand, the rote practising of these basic facts on their own, without any understanding, does not necessarily lead to permanent learning of the required facts.

The purpose of this paper is not to provide a complete program, but more to assist the teacher in structuring classroom activities to ensure that each child has the best chance of achieving the ready recall of basic number facts.

It is divided into three major parts corresponding to three possible stages of a child's progress towards recall of basic number facts:

- Stage A, the development of basic number facts;
- Stage B, the establishment of basic number facts; and
- Stage C, the memorisation of basic number facts.

Many articles have appeared recently supporting this sequence, and teachers are encouraged to follow up the references in the bibliography for a more detailed discussion than this paper contains.

It is not suggested that a child will move directly from Stage A to Stage B and then to Stage C. At different times, he/she could be expected perhaps to be working at different Stages in each of the process areas, e.g. a child could be working at Stage C in addition, Stage B in multiplication and Stage A in division.

Stage A

THE DEVELOPMENT OF BASIC NUMBER FACTS

During this Stage children will develop:

- the concepts of addition, subtraction, multiplication and division as an essential prerequisite to the development of basic number facts; and
- the ability to associate a basic fact with a concrete or pictorial model, and vice-versa.

Children have their first experiences through concrete objects which they can handle, move, and at the appropriate stage, talk about. It is very important that children should be given the practical personal experience from which they can form concepts, and that teachers should have a wide range of environmental and structured materials available to them in planning learning experiences for children. During this Stage, and also in Stages B and C to follow, the importance for children to verbalise what they are doing cannot be over-emphasised as an integral part of the learning process.

These experiences should be followed with actual objects, which children can represent by simple drawings. At a later time, abstract representations of the real world, for example a number line or mathematical symbols can take the place of the real objects.

Figure 1 (see pp. 4–5) presents diagrammatically various ways that basic addition and subtraction fact situations can be modelled using a range of different resources (such as concrete objectives, pictorial representation, structured material and number strips). Two different ways of looking at subtraction, 'take-away' and 'comparison' are also incorporated into the figure.

Figure 2 (see pp. 6–7) presents diagrammatically various ways that basic multiplication and division fact situations can be modelled and recorded. Two different ways of looking at division, sharing (partition) and measuring (quotition), are used.

The recording sections in Figures 1 and 2 are particularly important only during Stage B, although verbalisation will be occurring at both Stage A and Stage B.

It is important that children use a range of resources in developing an understanding of number facts, and each column in Figures 1 and 2 represents a different level of abstraction for the child. For example:

- Concrete objects – These should initially be the actual objects that are being discussed, e.g. pencils, sweets, toys. At a later Stage counters, bottle tops, stones may be acceptable to the child as a representation of the actual objects.
- Pictorial representation – Included here would be cardboard cut-outs, and flannel board figures, as well as the much more abstract drawing in a book or on the blackboard.
- Structured material – Children can derive enormous benefit through appropriate use of structured materials (such as Unifix, MAB and Cuisenaire) as these have been developed to represent certain mathematical relationships.
- Number strip, track or line – Some problems are more appropriately solved using a number strip, track or line. The number line may be seen by some children to be a more difficult working model than the number strip or track, and if this is the case, its introduction should be delayed.

Stage B

ESTABLISHING NUMBER FACTS

In Stage A, children were introduced to the concepts of addition, subtraction, multiplication and division through 'hands-on' experiences with concrete or pictorial materials. From these experiences, number facts begin to emerge. At first, the number facts would have been spoken. Then the child learns to record them with symbols in mathematical (number) sentences, e.g. $3 + 2 = 5$, $5 - 2 = 3$ etc. Whereas the addition and subtraction facts up to $9 + 9$ are fairly easily modelled, concretely or pictorially, the multiplication and division facts with products greater than say 18 or 20 are not so easily modelled. Hence other methods are often desirable to help in the establishment of such number facts prior to the final stage of memorisation, to be discussed in Stage C. This stage of establishing number facts is often referred to as the 'getting ready to memorise' stage.

Stage B is concerned largely with the development in the child of thinking strategies relating to number facts. Thinking strategies will generally rely on one or more of these:

- the finding of number patterns;
- the discovery and observation of relationships among number facts;
- the use of certain number properties;
- the verbalisation of what he or she is doing or thinking; and
- the ability to record with symbols the number facts which are spoken or thought.

Children's experiences with thinking strategies involving patterns, relationships and properties constitute a most important stage leading to meaningful memorisation of number facts.

Although there is no strict order in which a program may be sequenced, it is logical to deal with the easier number

Continued on page 8

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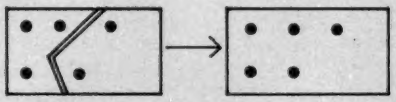
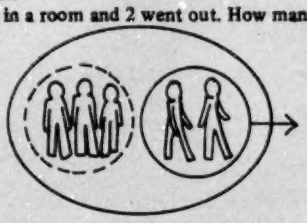
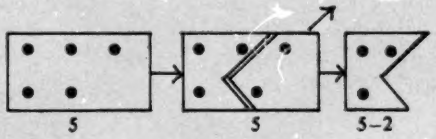
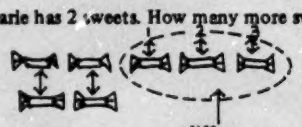
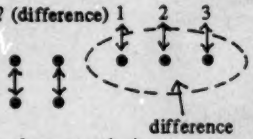
FIGURE 1: MODELS FOR ADDITION AND SUBTRACTION

		Set	Models
		Concrete materials Real objects, people, counters, flannel board cut-outs	Pictorial materials Representation of real objects
ADDITION		*Problem or Situation* Tom won 3 marbles yesterday and 2 today. How many does he have now? Model Recording 3 + 2 = 5	 3 + 2 = 5
SUBTRACTION	TAKE-AWAY	*Problem or Situation* There were 5 boys in a room and 2 went out. How many are left in the room? Model  Recording 5 – 2 = 3	 5 5 5–2 5 – 2 = 3
	COMPARISON	*Problem or Situation* John has 5 sweets and Marie has 2 sweets. How many more sweets has John than Marie? (difference) Model John Marie  difference difference = the number in the unmatched set Recording 5 – 2 = 3	 difference difference = the number in the unmatched set 5 – 2 = 3

Note on subtraction as missing addend (complementary addition).

Still another subtraction situation arises in a problem like: 'A boy has 2 marbles. How many more will he need so that he'll have 5 marbles altogether?'

Provided that children understand the meaning of 'how many more', and can also simultaneously visualise the subset of 2 marbles and the whole set of 5 marbles, they will be able to solve this problem. As it would seem to be a case of solving □ + 2 = 5 addition would be the operation to use. But since it is the missing addend 3 which has to be found, the problem is really a subtraction problem, and indeed it can be solved by considering the subtraction sentence 5 – 2 = □ . Hence 5 – 2 = 3 is related to 3 + 2 = 5.

The importance of this idea is referred to again when the inverse relationship between addition and subtraction is made use of in determining subtraction facts, i.e. 5 – 2 = 3 because 3 + 2 = 5.

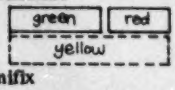
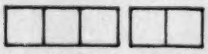
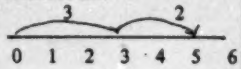
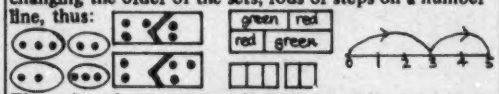
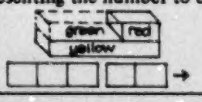
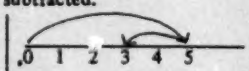
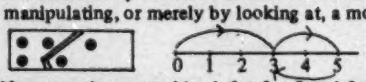
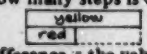
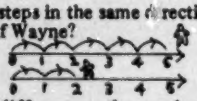
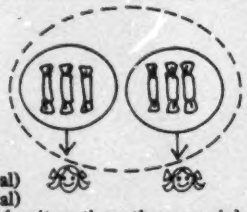
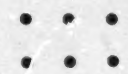
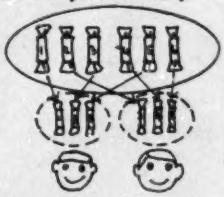
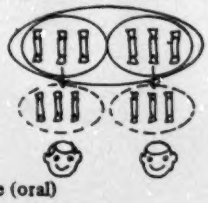
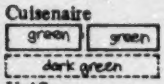
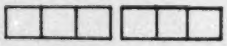
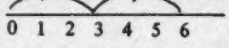
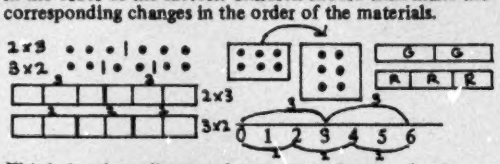
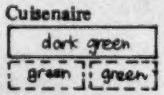
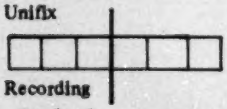
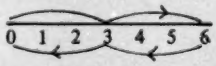
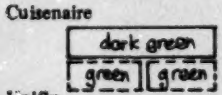
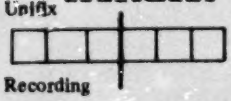
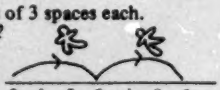
Measure	Models	
Structured aids		Comments
Cuisenaire, Unifix and similar aids	number strip or line or track	
Problem A boy walks 3 steps then 2 steps more. How many steps is he from his starting point? Cuisenaire  Unifix  3 + 2 = 5	Number line  3 + 2 = 5	*Commutative property of addition* Children should be given frequent exercises involving changing the order of the sets, rods or steps on a number line, thus:  Through such concrete or pictorial materials experiences, children should ideally discover for themselves that 2 + 3 = 3 + 2 and intuitively, that it makes no difference in what order two numbers are added; the sums are the same. Awareness of this property will always give children a second chance to obtain a number fact they cannot recall.
Problem I take 5 steps forward and 2 steps back. How many steps am I now from the start? We do not physically 'take away'. Using cuisenaire we 'cover up', and using a number line, we move back the part representing the number to be subtracted.  	 5 − 2 = 3	*Inverse relationships between addition and subtraction* By manipulating, or merely by looking at, a model such as:  children can then record both 3 + 2 = 5 and 5 − 2 = 3. They come to see that, starting with 3, adding 2, recording the result 5, and then subtracting 2, the final answer is the original number 3. Hence addition and subtraction of the same number produces no change in the original number. Whatever the addition of 2 did, the subtraction of 2 undid.
Problem Gary takes 5 steps and Wayne 2 steps in the same direction. How many steps is Gary ahead of Wayne?  difference = the value of the missing rod difference = the number of the unmatched subset. 5 − 2 = 3	 difference = the number of steps *not* taken by Wayne. 5 − 2 = 3	This illustrates the important relationships that exist between the operations of addition and subtraction which is called the inverse relationship because each reverses what the other does. Children do not need to be given the actual name 'inverse', but frequent experiences with material will give them a better basis for figuring out any subtraction fact such as 5 − 2 = 3 because they already know the corresponding addition fact 3 + 2 = 5. This will be discussed again in Stage B.

FIGURE 2: MODELS FOR MULTIPLICATION AND DIVISION

		Set	Models
		Concrete materials Real objects, people, counters, flannel board cut-outs	Pictorial materials Representation of real objects
MULTIPLICATION		*Problem or situation* Two girls get 3 sweets each. How many did they get altogether? Model  Recording 2 sets of 3 (oral) 2 lots of 3 (oral) 2 x 3 = 6 Read as 'two times three are six' or 'two threes are six'	*Problem* How many desks are there in 2 rows of 3 desks each? Model (array)  Recording 2 x 3 = 6
DIVISION	SHARING (PARTITION)	*Problem or situation* Six sweets are shared between 2 boys. How many does each boy get? Model  Recording 6 shared between 2 (oral) 6 divided by 2 = 3 sweets 6 ÷ 2 = 3	*Problem* If 6 boys are arranged in 2 rows, how many boys are there in each row? Model Recording 6 ÷ 2 = 3
	MEASURING (QUOTITION)	*Problem or situation* Six sweets are given out in groups of 3. How many boys get sweets? Model  Recording Six put into groups of three (oral) 6 ÷ 3 = 2 boys This answers the question 'How many 3's in 6?'	*Problem* Six boys are placed in rows of 3. How many rows will there be? Model Recording 6 ÷ 3 = 2 (rows) This answers the question 'How many 3's in 6?'

Measure	Models	
Structured aids		Comments
Cuisenaire, Unifix and similar aids	number strip or line or track	
Problem A boy makes 2 jumps of 3 spaces each along a line. How many spaces does he jump altogether? Cuisenaire  Unifix  Recording 2 x 3 = 6	 Recording 2 x 3 = 6	*Commutative property of multiplication* Exercises should be constantly structured to require a change in the order of the factors. Children should then make the corresponding changes in the order of the materials.  This helps them discover the property above – that it makes no difference in what order two factors are multiplied, the products are the same. Awareness of this property can help children to get a product at a later stage, e.g. 5 x 3 by already knowing 3 x 5.
Problem A boy jumps a total of 6 spaces in 2 equal jumps. How many spaces was each jump? Cuisenaire  Unifix  Recording 6 ÷ 2 = 3	 Recording 6 ÷ 2 = 3 This could be solved by 'trial and error' or by a folding technique to identify the '3'.	*Inverse relationship between multiplication and division* Materials may be used to demonstrate that if a number is multiplied by another number and then divided by the same number, the result is the original number, e.g. 2 x 3 = 6 and 6 ÷ 3 = 2 are corresponding sentences illustrated by  2 x 3 = 6 6 ÷ 3 = 2 Hence a child can learn to respond to a question like 6 ÷ 3 by asking 'How many 3's in 6?' Since he knows two 3's are six (2 x 3 = 6), then he can interpret 6 ÷ 3 = 2 in words – 'Six divided by three is two because two threes are six'.
Problem A boy jumps 6 spaces in jumps of 3 spaces each. How many jumps did he make? Cuisenaire  Unifix  Recording 6 ÷ 3 = 2 This answers the question 'How many 3's in 6?'	 This problem could also be solved by starting at '6' and subtracting. Recording 6 ÷ 3 = 2 This answers the question 'How many 3's in 6?'	

Continued from page 3

facts first and to organise the recording of them in some systematic and sequential way. The sequence used here is +, − to 5 and then to 10, +, −, x, ÷ to 18, x and ÷ to 81. Addition and subtraction will be developed together, and so will multiplication and division.

In particular, this Stage looks at how teachers can use:

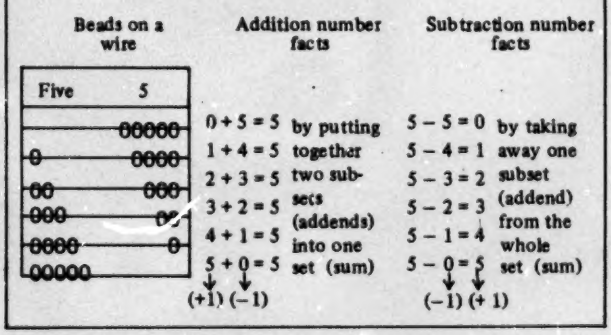
- the exploration of numbers to establish basic number facts;
- grids as a means of recording, organising, establishing and observing interrelations among basic number facts; and
- some special techniques to help a pupil work out multiplication facts.

Exploring numbers to establish basic number facts

Exploring numbers in terms of addition and subtraction

In Stage A, children would have modelled addition and subtraction situations. It is now possible to explore specific numbers using concrete materials as required. In the following example, '5' is explored.

FIGURE 3


Beads on a wire
Addition number facts
Subtraction number facts
Five 5
0 + 5 = 5 by putting
1 + 4 = 5 together
2 + 3 = 5 two sub-
3 + 2 = 5 sets
4 + 1 = 5 (addends)
5 + 0 = 5 into one set (sum)
5 − 5 = 0 by taking
5 − 4 = 1 away one
5 − 3 = 2 subset
5 − 2 = 3 (addend)
5 − 1 = 4 from the
5 − 0 = 5 whole set (sum)
(+1) (−1)
(−1) (+1)

Other aids suggested – Unifix cubes in two colours in value boats, two Cuisenaire rods matching a yellow rod.

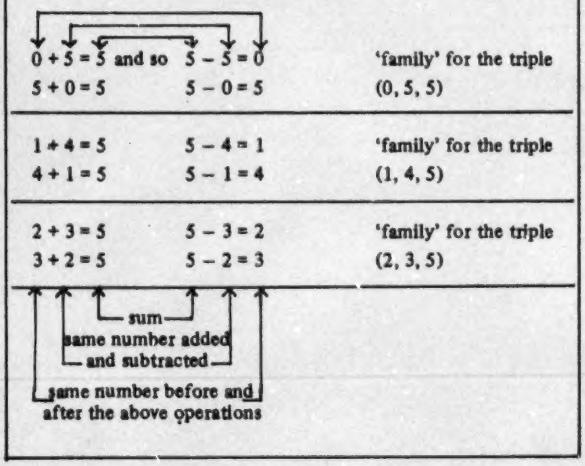
The six pairs of addition and corresponding subtraction facts shown in Figure 3 may also be regrouped into three 'families' of four number facts as in Figure 4.

Exploration of numbers carried out by children, using concrete or pictorial material followed by a recording of the results as shown above, provide bases for observation of number patterns, relationships and properties and the development of thinking strategies. Some of these, resulting from the exploration of the number 5 in Figure 4 are:

(i) *Number patterns*

The 'increasing by 1 (+1), decreasing by 1 (−1)' pattern in each pair of addends recorded in Figure 3 is important because it is the basis of the fundamental inverse relationship.

FIGURE 4


0 + 5 = 5 and so 5 − 5 = 0
5 + 0 = 5 5 − 0 = 5
'family' for the triple (0, 5, 5)
1 + 4 = 5 5 − 4 = 1
4 + 1 = 5 5 − 1 = 4
'family' for the triple (1, 4, 5)
2 + 3 = 5 5 − 3 = 2
3 + 2 = 5 5 − 2 = 3
'family' for the triple (2, 3, 5)
sum
same number added and subtracted
same number before and after the above operations

(ii) *Inverse relationship between the operations of addition and subtraction*

This is best explained by looking at each horizontal line in both Figures 3 and 4 where it is seen that starting with any number, the addition of any number followed by the subtraction of the same number results in:

(a) an addition sentence followed by a corresponding subtraction sentence; and

(b) no change in the number started with. This is because of the inverse relationship between addition and subtraction. The reverse is also true. A subtraction sentence followed by a corresponding addition sentence results in no change in the starting number.

(iii) *'Families' of number facts*

Figure 4 shows that the six addition and the six subtraction facts can also be grouped into three 'families' each of four number facts. They are called 'families' because each number fact in the family is related to the other three. A child could possibly work any number fact he or she does not know by referring to another member of the family which *is* known, e.g. 5 − 3 = 2 *because* 2 + 3 = 5. It will be noted that each family is derived from all of the addition and subtraction sentences that can be made from a consideration of the three triples (0, 5, 5), (1, 4, 5) and (2, 3, 5).

(iv) *Commutative property of addition*

Also illustrated in the 'families' is the application of the commutative property of addition. This means that it makes no difference in what order two numbers are added, the sums are the same. This could help a child to recall 7 + 9 so long as 9 + 7 were known.

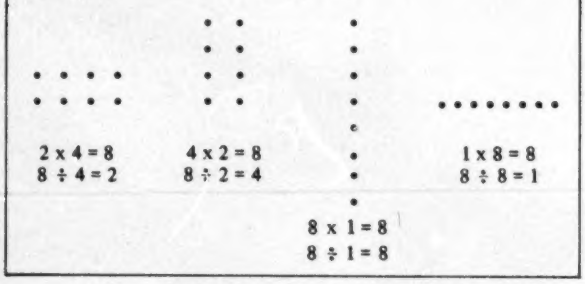
(v) *Special property of zero in addition*

This is demonstrated in Figures 3 and 4 where 5 + 0 = 0 + 5 because each is equal to 5. This illustrates the special property which 0 has in addition. Zero does *not* change the number to which it is added.

Exploring numbers in terms of multiplication and division

In Stage A, children would have modelled multiplication and division situations by arranging counters in arrays or in a single line. The following outline shows how a particular number can be explored to establish multiplication and division facts.

FIGURE 5



In each case, the corresponding division fact is shown with the multiplication fact. The four pairs of multiplication and division facts may also be arranged as shown in Figure 6.

FIGURE 6

Multiplication facts					Corresponding division facts				
Factor 1		Factor 2		Product	Product		Factor 2		Factor 1
1	x	8	=	8	÷	8	=	1	
2	x	4	=	8	÷	4	=	2	
4	x	2	=	8	÷	2	=	1	
8	x	1	=	8	÷	1	=	8	
↓		↓				↓		↓	
Doubling (x by 2)		Halving (÷ by 2)				Halving (÷ by 2)		Doubling (x by 2)	

During the exploration of a number in terms of multiplication and division, children will have had many opportunities to discover and observe number patterns, relationships and properties which may be put to practical use. Awareness of these should put children in a better position to establish number facts or to evolve thinking strategies to enable them to work out number facts which may, momentarily, have escaped recall. Some of these strategies are briefly discussed in the analysis below.

(i) *Number patterns*

The figure above shows a clear pattern of doubling (x by 2) and halving (÷ by 2) in the factors of the product. This is the basis of the important inverse relationship between multiplication and division.

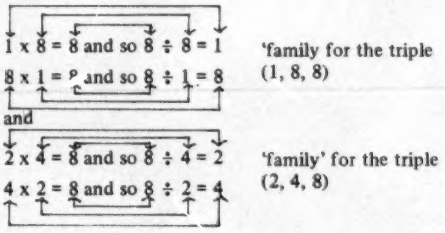
(ii) *Inverse relationship between multiplication and division*

On each horizontal line in Figure 6, it may be observed that a multiplication sentence (or fact) is followed by the corresponding division sentence (or fact), and that there is no change to a number if it is multiplied and then divided by the same number. This is because division is the inverse of multiplication. The opposite statement is also true. Multiplication reverses what division (by the same number) did.

(iii) *'Families' of multiplication and division facts*

The eight multiplication and division facts for the number 8 may also be grouped into two 'families' of four.

FIGURE 7



Awareness of the inverse relationship between multiplication and division, and the ability to think out all four members of a 'family' could be a great help to children to 'think out' any fact by referring to a corresponding fact or member of the same family, e.g. 10 ÷ 2 has to be 5 because 5 x 2 = 10. This would be evident to any child who can interpret 10 ÷ 2 as 'How many 2's in 10?' From knowing already that five 2's are 10 (5 x 2 = 10), the child will therefore be able to obtain the answer to 10 ÷ 2.

(iv) *Commutative property of multiplication*

This is similar to the commutative property of addition. Knowledge of the commutative property of multiplication will enable a child to 'work out', for example, two 5's only because five 2's is known as 10. Children should be given opportunities to discover these properties and also to find out whether the commutative property is true for the operations of subtraction and division.

(v) *Special property of 1 in multiplication*

As shown in Figure 6, 1 x 8 = 8 x 1 because each is equal to 8. This illustrates the special property which the number 1 has in multiplication, that if 1 is either factor, the product is equal to the other factor. Put another way, 1 does not change any number it multiplies or is multiplied by.

Use of grids

In the first instance, grids can provide a systematic way of recording number facts already experienced at the modelling stage, or discovered as a result of exploration of numbers in the previous section. Up till now, the number facts discussed have been mainly those with sums and products not exceeding 10. Further to this, various 'each-

ing methods for extending and rationalising number facts can be logically based on, or referred back to, a grid. This grid can be built up progressively by children either during or after the exploration of number facts. As mentioned earlier, the use of number patterns, number properties and relationships between numbers, operations and number facts would form part of any teaching strategy. As to how these may be aided by using and referring to grids is discussed below.

Use of addition-subtraction grid

FIGURE 8

Second Addend

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

First Addend

Doubles

The shaded area of the grid will have been already developed and partly established through the use of concrete and pictorial aids in Stage A. Children could then be given a variety of experiences leading to suitable ways of finding number facts with sums greater than 10 which they can record in the unshaded area. At this Stage, the use of concrete and pictorial materials would still be very relevant and practical. However, the teacher would now be asking children more and more to verbalise while each child is deciding which is his own most suitable method for finding particular number facts. The most suitable method will vary from child to child according to procedures with which he or she is familiar. Some methods which teachers use include:

(i) The sums 11 to 18 can very easily be filled in by the 'adding 1 on the previous sum' method in each horizontal line, e.g. $6 + 4 = 10$, $6 + 5 = 11$.

(ii) The method involving 'bridging the 10' can be handled by many children, particularly if they have already had adequate experience in the addition facts with sums of 10 through the exploration of that number, e.g.

$8 + 5$ $\quad 8 + (2 + 3)$
$\quad\quad\quad (8 + 2) + 3$
$\quad\quad\quad 10 + 3$
$\quad\quad\quad 13$

A child might verbalise this as 'eight, two more makes ten, and three are thirteen'.

This method illustrates the use of another property called the associative property of addition but it is the *use* of the property that is important and *not* its mathematical name.

(iii) Some number pairs seem to be easily remembered by children:
- (a) doubles: $5 + 5 = 10$, $7 + 7 = 14$ (as shown on the diagonal of the grid);
- (b) near doubles: $7 + 8 = 7 + 7 + 1 = 14 + 1 = 15$;
- (c) sharing numbers: $6 + 8 = 7 + 7 = 14$ (by adding 1 to one addend and subtracting 1 from the other as seen in the $(+1, -1)$ pattern in exploring 5 in Figure 4; and

(iv) 'Counting on': $8 + 5 = 9, 10, 11, 12, 13$.

Once completed, the grid can be used to observe the number patterns, properties and relationships inherent in it.

(i) *Number patterns*

Children should be encouraged to discover for themselves number patterns from the grids. There are many patterns on the addition grid in particular, and the time spent on this activity can contribute to the learning of number facts. One example would be to use the pattern of 13's and express each 13 as the sum of two addends. Since there are six 13's on the grid, there are six different addition facts observable directly from the grid: $9 + 4 = 13$, $8 + 5 = 13$, $7 + 6 = 13$, $6 + 7 = 13$, $5 + 8 = 13$, $4 + 9 = 13$.

Questions concerning the other combinations for 13, such as $10 + 3$, $11 + 2$ etc. not on the grid, could be used to stimulate discussion and reinforce number facts. The grid also shows a pattern of symmetry. If the grid were folded about the leading diagonal, running from the top left to the bottom right, the numbers on the left would coincide with the same numbers on the right. The same effect could be observed by holding a mirror vertically along this diagonal. The numbers on either side of the mirror that are reflected in the mirror are those on the opposite side of the grid.

(ii) *Properties*

On investigating the addends of any number of the grid and the 'mirror image' of that number, it will be discovered that $8 + 4 = 4 + 8$, $6 + 9 = 9 + 6$ etc. This is of course an illustration of the commutative property of addition, which has been referred to earlier.

(iii) *Relationships*

Along the diagonal itself, the numbers 0, 2, 4, 6, 8, 10, 12, 14, 16, 18 may be said to be reflections of themselves. These numbers are representations of the sums $0 + 0$, $1 + 1$, $2 + 2 \ldots 9 + 9$, and are therefore seen to be what has been referred to as 'doubles' Most children find these 'doubles' number facts fairly easy to memorise. It can also be demonstrated that because of the in-

verse relationship between addition and subtraction, the addition grid can be used to work out all subtraction facts, e.g. because 8 + 5 = 13, as ringed on the grid, the addend 5 above that can be subtracted from 13 to get the other addend 8. So 13 – 5 = 8 because 8 + 5 = 13. Frequent exercises like this can provide a child with experiences leading to establishment of number facts. It also provides a means of working out any subtraction fact by relating it to the corresponding addition fact which is probably already known.

Finally the grid may be used to establish all the 'families' of addition and subtraction facts, e.g.

6 + 9 = 15, 15 – 9 = 6	for the 'family' triple
9 + 6 = 15, 15 – 6 = 9	(6, 9, 15)

Children will also come to recognise that, in the case of triples like (7, 7, 14), there are only two members of the family, 7 + 7 = 14 and 14 – 7 = 7 and not the usual family of four facts. It is enough, and beneficial, for a child to realise that he has at least two, and generally four, related facts to call on to help in working out any fact that he wants to recall.

Use of multiplication-division grid

FIGURE 9

Second Factor

First Factor x	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Squares

The ways of using the multiplication and division grid (Figure 9) are similar to those discussed for the addition and subtraction grid. Children will observe and discover similar number patterns and patterns of symmetry which show up the commutative property of multiplication. They will also become aware of the fact that because of the inverse relationship between multiplication and division, the grid can be used as both a multiplication and a division grid.

Instead of the 'doubles' of addition, the leading diagonal shows the 'squares' of multiplication 0, 1, 4, 9, 16, 25, 36, 49, 64, 81. There will be 'families' of two related facts, one multiplication, one division, in the case of the square number triples such as (7, 7, 49), or of four related facts for other triples such as (7, 9, 63).

The fact that products are much larger numbers than those of the sums in addition makes it more difficult for children to establish multiplication facts with products greater than 18 and ranging up to 81. The stance taken in this paper is that division facts are no more difficult to arrive at than subtraction facts were if the inverse relationship method is the one used. The major problem then is to have as many meaningful ways of developing and establishing the multiplication facts.

Some special techniques to help a pupil work out multiplication facts

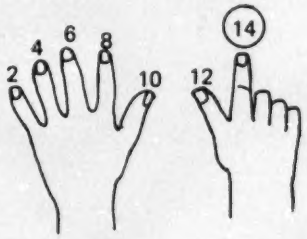
As well as those already discussed, further techniques which have been used by teachers and children are contained in this section. This does not imply that all, or some, techniques will be useful for all children, rather that some teachers have found them successful for particular children.

(i) Counting on in 2's, 3's or 5's (thinking in terms of sets of 2, 3 or 5)

seven twos 7 x 2

six threes 6 x 3

seven fives 7 x 5



(ii) Doubling – by adding (when 2 is the multiplier)

two nines 2 x 9 9 + 9 = 18

Note also the diagonal pattern of all the 'doubles' on the addition grid.

(iii) Trebling – by repeated adding (when 3 is the multiplier)

three sixes 3 x 6 → 6 + 6 + 6 → 12 + 6 = 18

(iv) Doubling and doubling (when 4 is the multiplier)

four eights 4 x 8

2 x 8 = 16, so 4 x 8 = 16 + 16, which is 2 tens + 12 ones, and this is 3 tens + 2 ones = 32

(v) Doubling and halving (usually when 5 is a factor)

seven fives 7 x 5

Since 5 is half of 10, then 7 x 5 is half of 7 x 10. Half of 7 tens is half of 6 tens and 1 ten, which is 3 tens and 5 = 35.

(vi) Doubling, doubling and doubling (when 8 is the multiplier)

eight sixes 8 x 6

one six double two sixes double four sixes double eight sixes

6 ⟶ 12 ⟶ 24 ⟶ 48

(vii) Trebling and doubling (when 6 is the multiplier)

six sevens 6 x 7

Since (by trebling) three sevens are 21, then (by doubling) six sevens are 21 + 21 = 42

(viii) Distributing (or sharing) the multiplication

(a) Distributing multiplication over addition

seven fives 7 x 5

seven fives is six fives and one five

30 + 5 = 35

or alternatively

four fives and three fives

20 + 15 = 35

(b) Distributing multiplication over subtraction (useful when 9 is a factor)

nine sevens 9 x 7 (or seven nines 7 x 9)

nine sevens is ten sevens less one seven 70 – 7 = 63

This shows how 63 is obtained.

This Stage, together with Stage A, has enabled children to develop and establish basic number facts. Many of these techniques require certain prerequisites, e.g. +n and –n facts to 10. For sums and products greater than 10, a pupil would need to have a good understanding of place value. The subsequent part of any mathematics program should allow children to practise these facts in a variety of situations so that memorisation will take place. The next Stage of this paper will help teachers to plan such a program.

Stage C

THE MEMORISATION OF BASIC NUMBER FACTS

For a child to be capable of using his basic number facts readily and accurately in order to handle efficiently a range of everyday situations, it is necessary for him to have memorised the answers to all combinations. Thus in a mathematics program, activities are provided which are designed to assist this memorisation. This stage of the paper is intended to help teachers design activities that will enable children to memorise more effectively basic number facts. These activities may be provided to assist the memorisation of a specific number fact (or group of related facts), or to give practice in applying a range of unrelated facts. As such they can range from oral recitation to playing a game.

As this paper has indicated, such memorisation activities take place after the child has been involved in Stages A and B aimed at developing and establishing the basic facts.

Although much incidental memorisation will have taken place during these Stages, their purpose was primarily to establish the necessary understandings on which meaningful learning is based. In doing so, the Stages were aimed at providing the child with ways of generating answers to the basic facts and with assisting memorisation through the organisation of the facts in a meaningful way.

The need exists, however, for a child to have the ability not only to generate answers, but also, and of equal importance, to recall with a minimum of delay, a specific answer. A child confronted with the need to recall the solution to 7 x 9 may not be in a situation which allows him time to search for and apply a strategy to generate the answer. What is often required is instant recall.

This recall comes about when a systematic and purposeful approach to the provision of drill activities is taken. Such an approach ensures that:

- practice is given in an area where a child has a specific need;
- the child is motivated to undertake the activity;
- monotony is overcome by the use of a wide variety of approaches; and
- the child is developing confidence in his use of the various basic number facts.

This section, then, provides a wide range of activities designed:

(a) to assist a child to memorise those facts which have not previously been incidentally memorised; and

(b) to give a child regular practice in using the basic combinations to ensure that previously memorised facts are frequently being recalled.

Such activities are often called drill or memorisation activities.

It is important to differentiate between drill and rote, two terms which are often used synonymously. The distinction can be made in terms of placement, rather than in tems of the type of activity used. Rote is seen as referring to learning which takes place when a child who does not have the required understandings to generate his own solution is provided with an activity which is aimed at establishing recall of the particular combination, e.g. repeating 7 x 9 = 63 a number of times. On the other hand, reciting 7 x 9 = 63 a number of times would be a drill activity if the child involved had indicated that he could produce a concrete model for the situation and could call on a strategy, e.g. 7 x 10 – 7 = 7 x 9; 7 x 3 x 3 = 7 x 9; 7 x 8 + 7 = 7 x 9, for finding a solution. Drill is memorisation following understanding. Rote is memorisation without understanding.

Activities to assist a child to establish immediate recall of basic number facts

Pupils find certain number facts difficult to recall. An approach to overcoming the difficulties that a pupil might

be having with a specific fact could involve the following steps:

(a) isolating facts which cannot be recalled;

(b) looking for a pattern among the facts that cannot be recalled, e.g.
- Are they associated with a particular family of facts?
- Are they associated with a particular number, e.g. addition facts which equal 15?
- Are they associated with a particular operation, e.g. multiplication by 8; subtraction of 7? and

(c) devising activities which provide the child with many opportunities to use the fact either alone or in association with related facts.

Should it be found that a pupil has weaknesses in many areas, this could be seen as an indication that insufficient time has been taken in the Stages when the facts were being developed or established. Hence it could be necessary to delay drill activities until such time as the child indicates a more complete understanding of how the facts are developed and established.

The following are a few examples of the type of activities which could be used.

Family of facts

(i) ● Use of flash cards – A child could prepare his own set of cards for facts which he has difficulty in recalling. He could be tested on these at regular intervals by the teacher or by another pupil. It is important that a pupil should have a record which indicates his areas of difficulty in the recall of basic number facts. This record could be in the form of a grid or a simple listing.

(ii) Completion type activities

● Fill in the boxes –
$8 + \square = 12$; $12 - 4 = \square$; $4 + 8 = \square$; $12 - 8 = \square$; $4 + \square = 12$; and $12 - \square = 3$.

● Make four number sentences using the numbers 4, 7, 28.

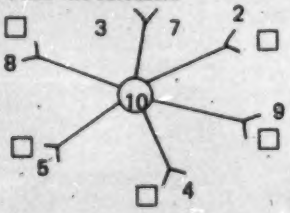
Facts for a particular number

(i) Completion-type activities

● Make it 12

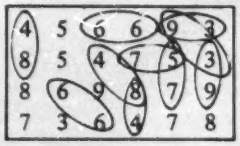
		9		8	
8	7		6		5

● Find the satellites



(ii) Puzzles

● Put a circle round adjacent pairs of numbers that add up to 12



(iii) Games

● Snap – A pack of cards with various number facts marked on them, e.g. (6 + 5), is provided. The game is played in pairs. Each player takes it in turn to play cards onto a centre pile. When a card is played which illustrates a combination for a designed number, e.g. 11, the player who recognises it wins the centre pile.

● Tens rummy – A pack of 40 cards with four cards each of the numbers 0 to 9 is provided. Ten cards are dealt to each player, and the remaining cards are turned down in the centre to make the draw pile. The top card is turned over to make the discard pile. The object of the game is to convert your own hand into five pairs of cards, the sum of each pair being 10. The game proceeds by selecting either from the draw pile or the discard pile, followed by the discarding of a card onto the discard pile. The first person to declare or show five pairs wins.

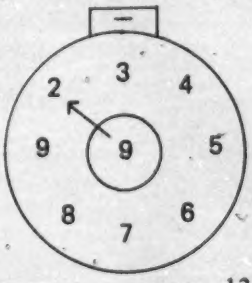
● Tens fish – A pack of 20 cards with two cards each of the numbers 0 to 9 is provided. All cards are turned over face down. In turn each player turns over two cards. If their sum equals 10, the player keeps the pair and has another turn. If the pair does not equal 10, the cards are replaced faced down. (Note for teachers: the game can be made easier or more difficult by decreasing or increasing the number of cards used.)

● Guess the number – This game is for three or more players. A bicycle spoke (or knitting needle) and five to 10 beads are needed. The dealer places a certain number of beads on the spoke, e.g. 8, and tells the number to the other players. He holds the beads at the top of the spoke and lets a certain number fall, e.g. 3, while keeping hidden the ones remaining. The first player to write down (or call out) correctly the number sentence (eight minus three equals five) that indicates the number of beads hidden, scores a point and takes a turn with the spoke. The first player to score five points wins.

(iv) Aids

● Spinner

Spin the arrow and write down the equation, e.g. $9 - 2 = 7$.



Facts associated with a particular operation

(i) Completion-type activities

	6	3	5	2	7	8
+4	10					

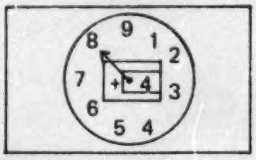
(ii) Puzzles

6 + 4	6+1	6+7	6+6	6+8	6+3	6+5	6+2

The player is provided with a number board or strip and eight cards. On the back of the cards marked 7 to 14 is printed a certain letter of the alphabet. The player places the cards on the strip so that the additions 6 + 4, 6 + 1 . . . are covered with the correct sum. When turned over the cards should spell out an eight letter word, e.g. 'addition'.

(iii) Aids

● Spinner



Spin the arrow and write down the equation, e.g. 4 + 8 = 12.

● Grid

18	72	28	33	27	14
7	64	36	30	48	6
60	25	32	15	21	70
4	20	18	12	63	60
16	9	54	8	35	81
24	40	50	42	22	56

Shade in the correct answers for this set of facts below:

4 x 3	4 x 6	4 x 4
4 x 9	4 x 5	4 x 7
4 x 2	4 x 8	

Check your answers by obtaining the 'fours' template from your teacher.

Activities to give a child regular practice in using basic facts

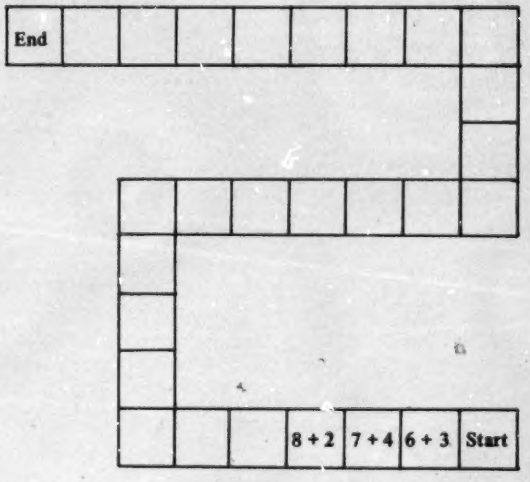
The activities outlined in the previous section were seen as providing a pupil with practice in recalling a particular fact or set of related facts to overcome an identified difficulty.

A primary child, no matter how well he may have mastered the basic number facts, will require planned practice sessions to help him maintain this mastery and develop it to a level where he can continually recall all facts accurately and readily. Such sessions should provide him with opportunities to use the facts in a variety of situations. These situations may vary from one in which he works by himself in solving a puzzle to one in which he competes against other children to win a game.

The following set of activities indicates that a varied approach to providing practice sessions can be adopted by a teacher. The set is a selection only of the wide range of activities which can be found in mathematics resource material today. Many of the ones provided here have been taken or adapted from the *Teachers Resource Book* for *Mathematics in Modules* on issue to schools, or from material on the *Annual Equipment and Materials Grants List*.

(i) Games

● Board game



Each player rolls a dice in turn. The number thrown sets the number of steps to proceed. On landing on a square, the player must answer the stated fact on that square. The first player to the end or beyond wins the game. An incorrect answer returns the player to the square from which he came.

In a variation, the games board has all the squares blank. The game is played with two dice, one marked with numbers such as 5 to 10, the other with directions such as +4, −3. The game proceeds with each child rolling the dice in turn. The answer to the problem set by the dice, e.g. 8 − 3 or 9 + 4, indicates the number of squares the child is to move. The first player to the end or beyond wins the game.

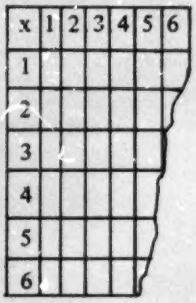
● Fast draw

This is a game for two people. A pack of 40 cards with the numbers comprising four sets each of ten

cards marked with numbers 0 to 9 is provided. The cards are shuffled and half are dealt to each player.

Both players place a card face up. The first to name the product (or sum) of the two numbers pick up both cards. If there is a tie, cards are left on the table and the next two cards played. The player who captures all the cards is the winner.

● Fill the grid



This game is for two, three or four players. A pack of cards which have numbers matching all the number fact products on the grid is provided.

The cards are turned face down and each player picks five cards. The first player selects one of his cards and places it on the correct square. The next player must select one of his cards which can can be placed so that it touches the card previously placed. As a card is played, or a player passes because he cannot play a card, another card is drawn by the player.

Note: The game can be scored and a winner found. – the player with the highest score, e.g. 1 for each card placed; 2 for completing a row, 3 for the first child to play all his cards.

● Find the triplets

The game is played with 36 cards – two for each of the numbers 1 to 18. All the cards are dealt to the players. The first player places a card with the number facing up. The second players places one beside it. The next player must put down a card which shows the sum or difference for the first two. If he cannot, it is the next player's turn. The player who completes the 'triplet' captures the three cards and places them in front of him. If no player has a sum or difference card, the player who put down the second cards puts both the cards in his hand. The game is over when a player is out of cards and no more 'triplets' can be made. The winner is the player with the most 'triplets'.

● Ready, set, shout

This game is played with the three dice provided. The first dice is numbered 3, 4, 5, 6, 7, 8. The second 4, 5, 6, 7, 8, 9 and the third has three multiplication signs, two addition signs and a subtraction sign. The object of the game is to score points by being the first player to correctly call out the answer to the stated operation on the two numbers when the dice are rolled. Players take it in turn to roll the three dice. One point is gained for each correctly stated answer, and one point lost for each wrong one. The first player to five points wins the game.

● ADDI bingo

The following rows of numbers are printed on a chart.

A	D	D	I
4	5	5	3
6	7	7	9
9	9	8	13
12	10	9	14
13	14	11	15
15	16	16	17
16	18	17	18

Each player selects four numbers from each column and enters them on his own ADDI Board, e.g.

A	D	D	I
6	10	5	3
9	14	9	13
12	16	11	17
13	18	17	18

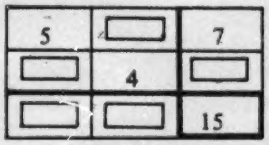
The banker, teacher or child, calls out a number fact and the column in which the answer should be found, e.g. 'under A, 4 + 8'. If a child has the correct answer, he covers it with a marker. The winning card is one with all answers covered. The winning card should be checked for possible incorrect answers.

Variations:
MULTI – Multiplication facts
MADS – Mixed facts, e.g.
A row multiplication
B row addition
C row division
D row subtraction

The winning card could also have only a row or a column filled, or an agreed upon number of squares filled.

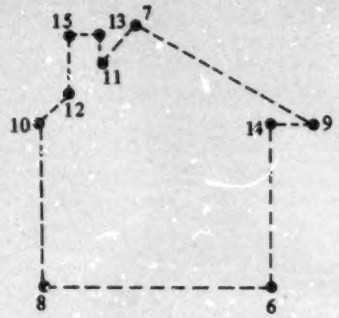
(ii) Puzzles

● Fill in the missing numbers



● Make a picture

Start 6 + 2
3 + 3
9 + 5
7 + 2
5 + 2
8 + 3
4 + 9
8 + 7
5 + 7
6 + 4
End 5 + 5



The numbers are joined in the order given to complete a picture.

● Magic Square

Complete the following magic square

	1	6
	5	7
	9	

Conclusion

This curriculum paper began with the premise that one desired outcome of a child's mathematics education is 'facility with accuracy in computation'. A child's ability readily to recall basic number facts was seen as an important early link in the chain leading to this outcome, and this paper has been devoted to the development, establishment and memorisation of these basic facts.

It should be remembered however, that the role of computational skills in mathematics must be seen in the light of the contributions they make to a child's ability to use mathematics in everyday living. This paper has made no reference to the basic mathematical skills that children need in order to function effectively in today's society. Teachers are encouraged to read the 'National Council of Supervisors of Mathematics Position Paper on Basic Skills' for information on this topic. Basic number facts will then be able to be seen as a prerequisite to, and therefore a vital integral part of, a mathematics program and not just as an end product in its own right.

December 1979

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All of the above books are available for loan in the Department of Education Library, 5th Floor, Estates House, 127 Creek Street, Brisbane, Q. 4000. A copy of the 'Position Paper on Basic Skills' is also available on request.

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